Surds and Indices

I. **Laws of Indices:**

i. $a^m * a^n = a^{m+n}$

ii. $a^m/a^n = a^{m-n}$
iii. $(a^m)^n = a^{mn}$
iv. $(a^b)^n = a^n b^n$
v. $(a/b)^n = a^n/b^n$
vi. $a^0 = 1$

II. **Surds:** Let a be a rational number and n be a positive integer such that $a^{1/n}$ = n√a is irrational. Then, na is called a surd of order n.

III. **Laws of Surds:**

i. $\sqrt[n]{a} = a^{1/n}$

ii. $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$

iii. $\sqrt[n]{a/b} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

iv. $\left(\sqrt[n]{a}\right)^n = a$

v. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

vi. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

1. To find $\sqrt{(a + \sqrt{b})}$ write it in the form $m + n + 2\sqrt{mn}$, such that $m + n = a$ and $4mn = b$, then $\sqrt{(a + \sqrt{b})} = \pm(\sqrt{m} + \sqrt{n})$

2. $(\sqrt{a}.\sqrt{a}.\sqrt{a}....\infty) = a$
3. If $(\sqrt{a} + \sqrt{a} + \sqrt{a}........\infty) = p$, then $p(p - 1) = a$.
4. If $a + \sqrt{b} = c + \sqrt{d}$, then $a = c$ and $b = d$.

Examples:
1. Simplify: (i) $(81)^{3/4}$ (ii) $(1/64)^{-5/6}$ (iii) $(256)^{-1/4}$

Solution:(i) $(81)^{3/4} = (3^4)^{3/4} = 3^3 = 27$.
(ii) $(1/64)^{-5/6} = 64^{5/6} = (2^6)^{5/6} = 2^5 = 32$

(iii) $(256)^{-1/4} = (1/256)^{1/4} = [(1/4)^4]^{1/4} = 1/4$

2. If $x=3+2\sqrt{2}$, then the value of $(\sqrt{x} - (1/\sqrt{x}))$ is:........
Solution:

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$$

$$= (3 + 2\sqrt{2}) + \frac{1}{(3 + 2\sqrt{2})} - 2$$

$$= (3 + 2\sqrt{2}) + \frac{1}{(3 + 2\sqrt{2})} \times \frac{(3 - 2\sqrt{2})}{(3 - 2\sqrt{2})} - 2$$

$$= (3 + 2\sqrt{2}) + (3 - 2\sqrt{2}) - 2$$

$$= 4.$$

$$\therefore \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = 2.$$

Exercise Questions

1.The value of $(\sqrt{8})^{1/3}$ is:
a.2
b. 4
c. 2
d. 8

Answer: Option c.
$(\sqrt{8})^{1/3} = (8^{1/2})^{1/3} = 8^{1/6} = (2^3)^{1/6} = 2^{1/2} = \sqrt{2}$.

2. The value of $5^{1/4} * (125)^{0.25}$ is:
a. √5
b.5√5
c.5
d.25
Answer: Option c
$5^{0.25} * (5^3)^{0.25} = 5^1 = 5$.

3. The value of $(32/243)^{-4/5}$ is:
a. 4/9
b. 9/4
c. 16/81
d. 81/16

Answer: Option d.
$(32/243)^{-4/5} = (243/32)^{4/5} = [(3/2)^5]^{4/5} = 81/16$

4. $(1/216)^{-2/3} \div (1/27)^{-4/3} = ?$
a. 3/4
b. 2/3
c. 4/9
d. 1/8
Answer: Option c.
$(1/216)^{-2/3} \div (1/27)^{-4/3} = 216^{2/3} \div 27^{4/3} = (63)^{2/3} \div (33)^{4/3} = 4/9$

5. $(2^{n+4} - 2.2^n)/(2.2^{n+3}) = 2^{-3}$ is equal to:
a. 2^{n+1}
b. $-2^{n+1} + 1/8$
c. $9/8 - 2^n$
d. 1
Answer: Option d.

$(2^{n+4} - 2.2^n)/2.2^{n+3} + 1/2^3 = 7/8 + 1/8 = 1$

6. If $5\sqrt{5} * 5^3 \div 5^{-3/2} = 5^{a+2}$, the value of a is:
a. 4
b. 5

c.6
d. 8

Answer: Option a
$5^{3/2} * 5^{3} \div 5^{-3/2} = 5^{a+2}$

$5^{3/2 + 3 + 3/2} = 5^{a+2}$
3/2 + 3 + 3/2 = a+2
a+2=6; a=4

7. If √2n =64, then the value of n is:
a. 2
b. 4
c. 6
d. 12

Answer: Option d
√2n =64 => $2^{n/2}$ = 64= 2^{6}
n/2=6; n=12

8.The simplified form of $(x^{7/2} /x^{5/2})$.√y3 /√y)is :
a.x^{2}/y
b. x^{3}/y^{2}
c. x^{6}/y^{3}
d. xy
Answer: Option d

$(x^{7/2} /x^{5/2})$. (√y3 /√y) = $x^{7/2 - 5/2}$. $y^{3/2 - 1/2}$ = xy

Exercise Questions

1. If *m* and *n* are whole numbers such that $m^{n} = 169$, then the value of $(m – 1)^{n + 1}$ is:

a. 1

b. 13

c. 169

d. 1728

2. The simplified form of $\frac{x^{9/2} . \sqrt{y^7}}{x^{7/2} . \sqrt{y^3}}$ is:

a. x^2/y^2

b. $x^2 . y^2$

c. xy

d. x^2/y

3. If $\sqrt{(3 + \sqrt[3]{x})} = 2$, then x is equal to :

a. 1

b. 2

c. 4

d. 8

4. If x is an integer, find the minimum value of x such that $0.00001154111 \times 10^x$ exceeds 1000.

a. 8

b. 1

c. 7

d. 6

5. Which among the following is the greatest?

a. 2^{3^2}

b. 2^{2^3}

c. $3^{2^\wedge 3}$

d. $3^{3^\wedge 3}$

6. Solve for *m* if $49(7^m) = 343^{3m + 6}$

a. -8/6

b. -2

c. -4/6

d. -1

7. Solve for $2^{y^\wedge \surd 2^\wedge 2} = 729$.

a. ±3

b. ±1

c. ±2

d. ±4

8. √[200√[200√[200……..∞]]] = ?

a. 200

b. 10

c. 1

d. 20

9. If a and b are positive numbers, $2^a = b^3$ and $b^a = 8$, find the value of a and b.

a. a = 2, b = 3

b. a = 3, b = 2

c. a = b = 3

d. a = b = 2

10. If $4^{4m+2} = 8^{6m-4}$, solve for *m*.

a. 7/4

b. 2

c. 4

d. 1

11. If 2^x x $16^{2/5} = 2^{1/5}$, then x is equal to:

a. 2/5

b. -2/5

c. 7/5

d. -7/5

12. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals :

a. xz/x + z

b. xz/2(x + z)

c. xz/2(x – z)

d. 2xz/(x + z)

13. If $7^a = 16807$, then the value of $7^{(a-3)}$ is:

a. 49

b. 343

c. 2401

d. 10807

14. If $3^x - 3^{x-1} = 18$, then the value of x^x is:

a. 3

b. 8

c. 27

d. 216

15. If $2^{(x-y)} = 8$ and $2^{(x+y)} = 32$, then x is equal to:

a. 0

b. 2

c. 4

d. 6

16. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is:

a. 0

b. 1

c. 1/abc

d. abc

17. 125 x 125 x 125 x 125 x 125 = $5^?$

a. 5

b. 3

c. 15

d. 2

18. If $5^{2n-1} = 1/(125^{n-3})$, then the value of n is:

a. 3

b. 2

c. 0

d. -2

19. If $x = 5 + 2\sqrt{6}$, then $\frac{(x - 1)}{\sqrt{x}}$ is equal to:

a. √2

b. 2√2

c. √3

d. 2√3

20. Number of prime factors in $\frac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}}$ is :

a. 56

b. 66

c. 112

d. None of these

Answer & Explanations

1. Exp: Clearly, m = 13 and n = 2.

Therefore, $(m – 1)^{n + 1} = (13 – 1)^3 = 12^3 = 1728$.

2. Exp: $\frac{x^{9/2} . \sqrt{y^5}}{x^{7/2} . \sqrt{y^3}}$ is: $= x^{(9/2 – 5/2)} . y^{(7/2 – 3/2)} = x^2. y^2$

3. Exp: On squaring both sides, we get:

$3 + \sqrt[3]{x} = 4$ or $\sqrt[3]{x} = 1$.

Cubing both sides, we get x = (1 x 1 x 1) = 1

4. Exp: Considering from the left if the decimal point is shifted by 8 places to the right, the number becomes 1154.111. Therefore, $0.00001154111 \times 10^x$ exceeds 1000 when x has a minimum value of 8.

5. Exp: $2^{3^\wedge2} = 2^9$

$2^{2^\wedge3} = 2^8$

$3^{2^\wedge3} = 3^8$

$3^{3^\wedge3} = 3^{27}$

As $3^{27} > 3^8$, $2^9 > 2^8$ and $3^{27} > 2^9$. Hence 3^{27} is the greatest among the four.

6. Exp: $49(7^m) = 343^{3m + 6}$ Þ $7^2 7^m$ Þ $(7^3)^{3m + 6}$ Þ $7^{2 + m} = 7^{9m + 18}$

Equating powers of 7 on both sides,

m + 2 = 9m + 18

-16 = 8m Þ m = -2.

7. Exp: $3^{y^\wedge\sqrt{2}^\wedge2} = 729$

$3^{y^\wedge2} = 3^4$ $(\sqrt{2}^2 = (2^{1/2})^2 = 2)$

equating powers of 2 on both sides,

$y^2 = 4$ Þ $y = \pm2$

8. Exp: Let √[200√[200√[200……..∞]]] = x ; Hence √200x = x

Squaring both sides 200x = x^2 Þ x (x – 200) = 0

Þ x = 0 or x – 200 = 0 i.e. x = 200

As x cannot be 0, x = 200.

9. Exp: $2^a = b^3$ ….(1)

$b^a = 8$ …..(2)

cubing both sides of equation (2), $(b^a)^3 = 8^3$

$b^{3a} = (b^3)^a = 512$.

from (1), $(2^a)^a = (2^3)^3$.

comparing both sides, a = 3

substituting a in (1), b =2.

10. Exp: $4^{4m + 2} = (2^3)^{6m - 4}$ => $4^{4m + 2} = 2^{18m - 12}$

Equating powers of 2 both sides,

4m + 2 = 18m – 12 => 14 = 14m => m = 1.

11. Exp: $2^x \times 16^{2/5} = 2^{1/5}$

=> $2^x \times (2^4)^{2/5} = 2^{1/5 =>}$ $2^x \times 2^{8/5} = 2^{1/5}$.

=> $2^{(x + 8/5)} = 2^{1/5}$

=> x + 8/5 = 1/5 => x = (1/5 – 8/5) = -7/5.

12. Exp: Let $a^x = b^y = c^z = k$. Then, $a = k^{1/x}$, $b = k^{1/y}$, $c = k^{1/z}$.

Therefore, $b^2 = ac$ => $(k^{1/y})^2 = k^{1/x} \times k^{1/z}$ => $k^{2/y} = k^{(1/x + 1/z)}$

Therefore, 2/y = (x + z)/xz => y/2 = xz/(x + z) => y = 2xz/(x + z).

13. Exp: $7^a = 16807$, => $7^a = 7^5$, a = 5.

Therefore, $7^{(a - 3)} = 7^{(5 - 3)} = 7^2 = 49$.

14. Exp: $3^x - 3^{x - 1} = 18$ => $3^{x - 1}(3 - 1) = 18$ => $3^{x - 1} = 9 = 3^2$ => x – 1 = 2 => x = 3.

15. Exp: $2^{(x - y)} = 8 = 2^3$ => x – y = 3 ---(1)

$2^{(x+y)} = 32 = 2^5 \Rightarrow x + y = 5$ ---(2)

On solving (1) & (2), we get x= 4.

16. Exp: $a^1 = c^z = (b^y)^z = b^{yz} = (a^x)^{yz} = a^{xyz}$. Therefore, xyz = 1.

17. Exp: 125 x 125 x 125 x 125 x 125 = $(5^3 \times 5^3 \times 5^3 \times 5^3 \times 5^3) = 5^{(3+3+3+3+3)} = 5^{15}$.

18. Exp: $5^{2n-1} = 1/(125^{n-3}) \Rightarrow 5^{2n-1} = 1/[(5^3)^{n-3}] = 1/[5^{(3n-9)}] = 5^{(9-3n)}$.

$\Rightarrow 2n - 1 = 9 - 3n \Rightarrow 5n = 10 \Rightarrow n = 2$.

19. Exp: $x = 5 + 2\sqrt{6} = 3 + 2 + 2\sqrt{6} = (\sqrt{3})^2 + (\sqrt{2})^2 + 2 \times \sqrt{3} \times \sqrt{2} = (\sqrt{3} + \sqrt{2})^2$

Also, $(x - 1) = 4 + 2\sqrt{6} = 2(2 + \sqrt{6}) = 2\sqrt{2}\,(\sqrt{2} + \sqrt{3})$.

Therefore, $\frac{(x-1)}{\sqrt{x}} = \frac{2\sqrt{2}\,(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = 2\sqrt{2}$.

20. Exp: $\frac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}} = \frac{(2 \times 3)^{12} \times (5 \times 7)^{28} \times (3 \times 5)^{16}}{(2 \times 7)^{12} \times (3 \times 7)^{11}} =$

$= \frac{2^{12} \times 3^{12} \times 5^{28} \times 7^{28} \times 3^{16} \times 5^{16}}{2^{12} \times 7^{12} \times 3^{11} \times 7^{11}} = 2^{(12-12)} \times 3^{(12+16-11)} \times 5^{(28+16)} \times 7^{(28-12-11)}$

$= 2^0 \times 3^{17} \times 5^{44} \times 7^{-5} = \frac{3^{17} \times 5^{44}}{7^5}$

Number of prime factors = 17 + 44 + 5 = 66.